in Fig. 3, where the twist induced drag of two wings of aspect ratio 20 having different twists are plotted. As compared to the continuous twist, the reversed twist is by far superior at the lower lift coefficients, that is, at higher speeds. For lower aspect ratios these differences become less pronounced.

The additional induced drag of such special twist types cannot, however, be satisfactorily calculated by the K factors given in this note, and instead, requires a special computation,

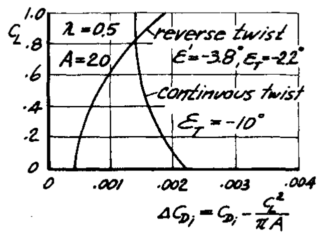

Fig. 3 Comparison of twist induced drag of a continuous and a reversed twist distribution for a trapezoidal wing.

using, e.g., Ref. 3. The outboard twist function must, of course, be optimized, such as to yield the desired drag characteristics. The resulting discontinuities in the wing skin are preferably buried in a nacelle, or if not available, locally faired.

Summary

Wing twist is primarily applied for two reasons: 1) to prevent the local onset of stall over some portion of the wing and 2) to achieve a minimum of induced drag at some optimum lift coefficient for cruise flight. Inspection of Eq. (2) reveals that induced drag consists of the following three interacting elements: lift, planform and section shape, and twist.

It was pointed out that the twist equation [Eq. (1)] has distinct advantages in wing design and manufacture, owing to the resulting linear leading and trailing edges. Twist induced drag coefficients determined by Eqs. (1) and (2), as compared to those of Ref. 2 (linear twist), are of lesser magnitude. Locally reversed or constant twist offers opportunities to minimize the twist induced drag for certain values of lift coefficient (i.e., speed), as shown in Fig. 3. The corresponding optimum twist distribution and induced drag can be found only by special computations. It should be noted that the differences in induced drag between wings of zero, continuous, and/or reversed twist become less pronounced as aspect ratios decrease.

The span efficiency "e" factor in the first term of Eq. (2) should be estimated preferably by the method of Ref. 7 which accounts for effects of Reynolds number. Wing theory, on the other hand, yields the well-known Glauert correction factor (Ref. 6, pp. 205, 215). It is suggested that the method given in Ref. 1 be replaced by that of Ref. 3 and this note.

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Multitapered Wings

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Nomenclature

W_1 = weight of outer panel, lb
W_2 = weight of inner panel, lb
W = total weight, lb
W_g = design gross weight, lb
N = load factor
ρ = material density, lb/in.^{3}
p = pressure, psi
M_1 = bending moment at station y in outer panel, in.-lb
M_2 = bending moment at station y in inner panel, in.-lb
h = maximum depth of section at station y, in.
k = ratio of effective depth to maximum depth
h_B = maximum depth at break station, in.
h_T = maximum depth at tip, in.
h_R = maximum depth at root, in.
c_R = root chord, in.
C_T = tip chord, in.
c_B = chord at break, in.
λ_1 = taper ratio of outer panel
λ_2 = taper ratio of inner panel
$f(\lambda)$ = taper ratio function $= \frac{1}{3} + [\lambda/(1-\lambda)] - 2[\lambda/(1-\lambda)]^2 + 2[\lambda/(1-\lambda)]^3 \log(1/\lambda)$
S_1 = area of outer panel, in.^{2}
S_2 = area of inner panel, in.^{2}
AR_1 = aspect ratio of outer panel
AR_2 = aspect ratio of inner panel
S = total wing area, in.^{2}
Λ = sweep of the 50% chord, deg
W_{sec} = weight of secondary structure, lb
ϕ = $(c_B - C_T)/e$
ψ = $c_R - C_B/a$
f = average working stress, psi

In most theoretical analyses of wings it is assumed that the wing has constant planform taper. However, there are many examples in the design of aircraft in which it has been found advantageous to employ a wing composed of sections of different rates of taper. In Ref. 1 a theoretical wing weight was obtained for wings of constant planform taper and of constant thickness ratio. An expression for the weight of a wing consisting of two sections of different planform taper ratios is developed in this paper (Fig. 1), and it is clear that the method can be extended to obtain the weights of wings of more than two tapers. Considering the outer panel and assuming a uniform pressure distribution, the bending moment

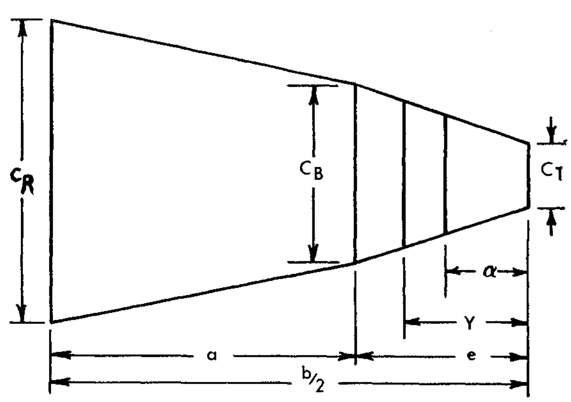

Fig. 1 Weight of wing consisting of two sections of different planform tapers.

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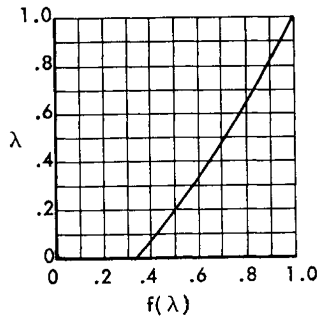

Fig. 2 Plot of $f(\lambda)$ vs λ.

at an arbitrary station y is obtained by

$$M_1 = \int_0^y p(c_T + \phi\alpha)(y - \alpha)d\alpha = p\left[\frac{C_T y^2}{2} + \frac{\phi y^3}{6}\right] \quad (1)$$

and the maximum depth of the wing at station y is given by

$$h = h_B\lambda_1 + h_B(1 - \lambda_1)y/e = h_T + \beta y \quad (2)$$

and where

$$\beta = [h_B(1 - \lambda_1)]/e \quad (3)$$

The weight of the outer panel can be written

$$W_1 = \rho\int_0^e A dy = \frac{\rho}{kf}\int_0^e \frac{M_1}{h} dy \quad (4)$$

Substituting the expressions given by Eqs. (1–3) into Eq. (4)

$$W_1 = \frac{p\rho}{6kf}\int_0^e \frac{(3C_T y^2 + \phi y^3)}{h_T + \beta y} dy = \frac{p\rho}{6kf}\left\{\frac{3C_T e^3}{h_B^3(1 - \lambda_1)^3}\left[-h_B(1 - \lambda_1)h_T + \frac{h_B^2(1 - \lambda_1)^2}{2} + h_T^2 \log\frac{h_B}{h_T}\right] + \frac{e^4(c_B - c_T)}{h_B e}\left[\frac{1}{3(1 - \lambda_1)} - \frac{\lambda_1}{2(1 - \lambda_1)^2} + \frac{\lambda_1^2}{(1 - \lambda_1)^3} + \frac{\lambda_1^3}{(1 - \lambda_1)^4}\log\lambda_1\right]\right\} \quad (5)$$

Simplifying in Eq. (5)

$$W_1 = \frac{3p\rho}{6kf h_B/C_B}\left[\frac{1}{9} + \frac{\lambda_1}{3(1 - \lambda_1)} - \frac{2\lambda_1^2}{3(1 - \lambda_1)^2} + \frac{2\lambda_1^3}{3(1 - \lambda_1)^3}\log\frac{1}{\lambda_1}\right] \quad (6)$$

Considering the inboard panel, the bending moment at an arbitrary station y is obtained by

$$M_2 = M_e + p\int_0^y (c_B + \psi\alpha)(y - \alpha)d\alpha = M_e + \frac{p(3c_B y^2 + \psi y^3)}{6} \quad (7)$$

where

$$M_e = p[(c_T e^2/2) + (\phi e^3/6)] \quad (8)$$

and the maximum depth is given by

$$h = h_R\lambda_2 + h_R(1 - \lambda_2)(y/a) = h_B + Ey \quad (9)$$

Similarly, the weight of the inboard panel is derived from

$$W_2 = \rho\int_0^a A dy = \frac{\rho}{kf}\int_0^a \frac{M_2 dy}{h} = \frac{p\rho}{6kf}\int_0^a \frac{3c_B y^2 + \psi y^3}{h_B + Ey} dy + \frac{\rho}{kf}\int_0^a \frac{M_e dy}{h_B + Ey} = \frac{M_e\rho}{kfE}\log\frac{h_R}{h_B} + \frac{3p\rho a^3}{6kf h_R/c_R}\left[\frac{1}{9} + \frac{\lambda_2}{3(1 + \lambda_1)} - \frac{2\lambda_2^2}{3(1 - \lambda_2)^2} + \frac{2\lambda_2^3}{3(1 - \lambda_2)^3}\log\frac{1}{\lambda_2}\right] \quad (10)$$

Noting that $p = W_g N/S$, Eqs. (6) and (10) are combined to obtain the total weight of the wing:

$$W = W_1 + W_2 = \frac{W_g N e^2 a\rho \log(1/\lambda^2)}{kfSh_B/c_B[(1/\lambda_2) - 1]}\left(\frac{\lambda_1}{2} + \frac{1 - \lambda_1}{6}\right) + \frac{W_g N\rho}{2kfS}\left\{\frac{e^3}{h_B/c_B}\left[\frac{1}{9} + \frac{\lambda_1}{3(1 - \lambda_1)} - \frac{2\lambda_1^2}{3(1 - \lambda_1)^2} + \frac{2\lambda_1^3}{3(1 - \lambda_1)^3}\log\frac{1}{\lambda_1}\right] + \frac{a^3}{h_R/c_R}\left[\frac{1}{9} + \frac{\lambda_2}{3(1 - \lambda_2)} - \frac{2\lambda_2^2}{3(1 - \lambda_2)^2} + \frac{2\lambda_2^3}{3(1 - \lambda_2)^3}\log\frac{1}{\lambda_2}\right]\right\} \quad (11)$$

Equation (11) can be further simplified:

$$W = \frac{W_g N\rho e^2 a \log(1/\lambda_2)}{kfSh_B/c_B[(1/\lambda_2) - 1]}\left(\frac{2\lambda_1 + 1}{6}\right) + \frac{W_g N\rho}{6kfS}\left\{\frac{e^3}{h_B/c_B}f(\lambda_1) + \frac{a^3}{h_R/c_R}f(\lambda_2)\right\} \quad (12)$$

A plot of $f(\lambda)$ vs λ is given in Fig. 2. Defining $AR_1 = e^2/S_1$ and $AR_2 = a^2/S_2$, Eq. (12) can be expressed in terms of the basic geometric terminology pertaining to wings:

$$W = \frac{W_g N\rho\lambda_2}{kfSh_B/C_B}\left[\frac{AR_1 S_1 (AR_2)^{1/2}(S_2)^{1/2}\log(1/\lambda_2)}{1 - \lambda_2}\right] \times \left(\frac{2\lambda_1 + 1}{6}\right) + \frac{W_g N\rho}{6kfS}\left\{\frac{AR_1^{3/2}S_1^{3/2}}{h_B/c_B}f(\lambda_1) + \frac{AR_2^{3/2}S_2^{3/2}}{h_R/C_R}f(\lambda_2)\right\} \quad (13)$$

Note that

$$\lim_{\lambda\to 1}\frac{\log(1/\lambda)}{1 - \lambda} = 1$$

It is to be noted that $f(\lambda)$ is a monotonic increasing function of λ.

For a wing of a single constant taper, $S_2 = 0$, and Eq. (13) reduces to

$$W = [W_g N\rho S^{1/2}AR^{3/2}f(\lambda)/48kf(h/c)] \quad (14)$$

If all factors in Eq. (14) are constant except area

$$W \alpha S^{1/2} \quad (15)$$

and

$$W_{\sec}\alpha S \quad (16)$$

If the span and absolute maximum thickness distribution are constant, we substitute

$$AR = b^2/S \quad (17)$$

and

$$c_{\text{ave}} = S/b \quad (18)$$

into Eq. (14) and obtain

$$W = [W_g N\rho b^2 f(\lambda)/48kfh] \quad (19)$$

which indicates that the weight of the box beam is independent of area. If the span and thickness ratio are constant, we substitute Eq. (17) into Eq. (14) and obtain

$$W = [W_g N\rho b^3 f(\lambda)/48kfS(h/c)] \quad (20)$$

Equations (19) and (20) illustrate the high sensitivity of wing weight to span. It is concluded that large wing areas can be obtained for a relatively small wing weight by maintaining a minimum span, constant thickness or thickness ratio, and increasing the chord.

The effect of sweep may be taken into account by incorporating the factor $\sec\Lambda$ into Eq. (14).

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